Evaluation of Boltzmann Integrals for Inelastic Collision and Radiative Processes in Monatomic Plasmas *

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Within the scope of a moment method collision integrals of the Boltzmann type are formulated and evaluated for the different excited particle states of a stationary monatomic plasma macroscopically at rest. All collision and radiative processes except the three body recombination are considered. The calculations are performed within Grad's 8-moment approximation of the momentum distribution functions of the material particle components. Assuming axial symmetry of the differential collision cross sections the evaluation leads to linear functions of generalized Chapman-Cowling integrals for binary inelastic particle interactions and to Einstein and photoionization coefficients for the radiative processes. Simplifications are given in special and limiting cases.

I. Introduction

Generalizations of the transport theory of gases and gas mixtures including inelastic collision processes have been worked out by a number of authors [1–8]. The investigations quoted either use the Chapman-Enskog technique or the moment equations method.

Concerning the moment equations method, which we also adopt in this paper, there are two different approaches:

The one approach proposed by McCORMACK [6], ALIESVKII and ZHDANOV [7] is based on a double series expansion of the distribution funtions of the various particle species (electrons, ions, different kinds of neutrals) in the translational and internal particle states using irreducible Hermite tensor polynomials of the velocity and orthogonal polynomials of the internal energy as introduced by CHANG and UHLENBECK [1]. The double series expansion is an extension of GRAD's [9] series expansion including the internal energy as a discrete variable. Consequently its coefficients present themselves as weighted sums of velocity moments over all internal energy states. Since only these averaged moments enter into the expansion, one needs for each moment only one equation for each particle species. Note, however, that due to the internal degrees of freedom additional moments have to be considered. If, for instance, internal heat flux and internal temperature are included, the 13-moment approximation of the distribution functions passes over into the 17-moment approximation, which has been applied to transport and relaxation problems in polyatomic gases with rotational and vibrational degrees of freedom.

The other approach proposed by BYDDER and LILEY [8] treats all internal energy states as separate components of the mixture. It produces a set of coupled moment equations for all excited states of the particle species. For each moment one has as many equations as there are different internal energy states. Since under these circumstances the distribution functions are expanded in velocity space only, one returns to Grad's Hermite tensor polynomial expansion. Within the 13-moment approximation of the distribution functions this approach has been applied to the transport theory of

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thermal nonequilibrium partially ionized plasmas main attention being directed to inelastic collision processes of material particles.

The present work is an extension of the second approach in so far as it includes all relevant collision and radiative processes in monatomic plasmas with the exception of the three body recombination. By a consequent use of the microscopic reversibility concept the collision operators and their velocity moments are presented as concisely as possible. In view of the application to transport phenomena in quiescent plasmas the 8-moment (hydrostatic pressure) approximation seemed to be appropriate.

Section II contains a short formulation of the dynamics of binary inelastic and superelastic particle collisions. It includes some transformations useful for the evaluation of the collision integrals accounting for thermal nonequilibrium between different particle species. In section III we give a closed set of moment equations for the various excited particle states within the 8-moment approximation. In section IV the collision operators in the kinetic equations of the excited neutral atoms are formulated for the collision and radiative processes under consideration. Taking the velocity moments of the collision operators collision integrals are derived, which by means of some integral transformations are reduced to generalized Chapman-Cowling integrals as well as to photoionization and Einstein coefficients for the radiative processes. The collision integrals for the general case of binary particle collisions are summarized in section V. In section VI we examine simplifications that result for the collision integrals in special and limiting cases. Finally, a discussion of the results is presented in section VII.

II. Equations for the Binary Inelastic Particle Encounter

We describe within an unified formulation all types of binary elastic, inelastic, and superelastic collision processes

$$A_i + \bar{A}_k \rightleftharpoons A'_j + \bar{A}'_l ,$$

in which an excited neutral atom A_i of species A in internal energy state i takes part. $\bar{A}_k$ may alternatively be considered as a free electron e_-, as an ion A_+ in its internal energy ground state, or as an excited neutral atom of a second species $\bar{A}$ in internal energy state k. Primed quantities indicate the final momentum state of the collision partners.

Let us characterize the particle component A_i by its momentum $\boldsymbol{p}_i = m_i \boldsymbol{v}_i$ and its internal energy ε_i designating corresponding quantities of $\bar{A}_k$ by a bar. Then, by introduction of relative and center-of-mass velocities

$$\boldsymbol{g}_{ik} = \boldsymbol{v}_i - \bar{\boldsymbol{v}}_k ; \quad \boldsymbol{G}_{ik} = M_i \boldsymbol{v}_i + \bar{M}_k \boldsymbol{v}_k \tag{1}$$

we get the energy and momentum equations

$$g'^2_{jl} = R^2 g^2_{ik} , \quad \boldsymbol{G}'_{jl} = \boldsymbol{G}_{ik} . \tag{2}$$

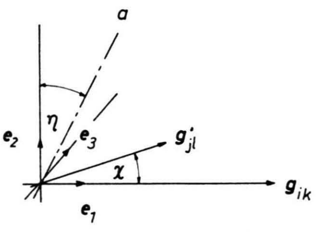
The quantity

$$R^2 - 1 = (2 \Delta_{ikjl} / \mu g^2_{ik}) \ \text{with} \ \Delta_{ikjl} = \varepsilon_{ik} - \varepsilon_{jl} \tag{3}$$

can be interpreted as inelastic energy loss parameter, where $\varepsilon_{ik} = \varepsilon_i + \bar{\varepsilon}_k$ holds and $\mu = m_i \bar{m}_k / (m_i + \bar{m}_k)$ is the reduced mass. The vector diagram for the relative velocities in the initial and final state of the collision partners is shown in the following figure, from which the relations

$$\begin{aligned} \boldsymbol{g}_{ik} &= g_{ik} \boldsymbol{e}_1 , \\ \boldsymbol{g}'_{jl} &= R\, g_{ik} [\cos \chi\, \boldsymbol{e}_1 + \sin \chi (\cos \eta\, \boldsymbol{e}_2 + \sin \eta\, \boldsymbol{e}_3)] \end{aligned} \tag{4}$$

can be read.



χ, η signify the scattering and the azimuthal angle of the particle orbit in the center-of-mass system. a is the intersection line between the orbit plane and the plane stretched by the unit vectors $\boldsymbol{e}_2$ and $\boldsymbol{e}_3$.

To evaluate the Boltzmann collision integrals for plasma components with different translational temperatures the transformation formulae

$$\boldsymbol{\zeta}_{ik} = \sqrt{\gamma}\, \boldsymbol{g}_{ik}, \ \boldsymbol{V}_{ik} = \sqrt{\beta_i + \bar{\beta}_k}\, [\boldsymbol{G}_{ik} + (\bar{M}_k - \bar{\Gamma}_k)\, \boldsymbol{g}_{ik}] \tag{5}$$

are useful so that we have

$$\mathrm{d}^3 p_i\, \mathrm{d}^3 \bar{p}_k = \left[\frac{m_i \bar{m}_k}{\sqrt{\beta_i \bar{\beta}_k}} \right]^3 \mathrm{d}^3 V_{ik}\, \mathrm{d}^3 \zeta_{ik} . \tag{6}$$

Here we introduced the inverse square of the most probable relative velocity $\gamma = \beta_i \bar{\beta}_k / (\beta_i + \bar{\beta}_k)$ with $\beta_i = m_i / 2\, k\, T_i$ and $\bar{\beta}_k = \bar{m}_k / 2\, k\, \bar{T}_k$. Moreover the abbreviations $\mu = m_i \bar{M}_k = \bar{m}_k M_i$ and $\gamma = \beta_i \bar{\Gamma}_k$

$=\bar{\beta}_k \Gamma_i$ have been used. It is obvious that for the present case $m_i = m$ and $\bar{m}_k = \bar{m}$. In the following we further assume that $T_i = T$ and $\bar{T}_k = \bar{T}$.

III. Moment Equations

The starting point for the derivation of transport relations in steady state plasmas are the kinetic equations

$$\frac{\boldsymbol{p}_\alpha}{m_\alpha}\frac{\partial f_\alpha}{\partial \boldsymbol{x}} + \boldsymbol{F}_\alpha \frac{\partial f_\alpha}{\partial \boldsymbol{p}_\alpha} = \frac{\partial_s f_\alpha}{\partial t} \tag{7}$$

for the material particle components α, and the photon Boltzmann equation. The material particle components comprise all different excited atomic particle states A_i as well as components $\bar{A}_k$, which include free electrons e_-, ions A_+ in their internal energy ground state, and excited states of a second atomic particle species, if such exists. $\partial_s f_\alpha/\partial t$ is the collision operator. The only forces to be considered here are of electrostatic origin so that $\boldsymbol{F}_\alpha = e_\alpha \boldsymbol{E}$, where $\boldsymbol{E}$ is the sum of external and selfconsistent electric field. m_α, e_α signify respectively mass and charge of the component α.

A standard method produces from Eq. (7) the system of moment equations

$$\frac{\partial}{\partial \boldsymbol{x}}\left[n_\alpha \langle \boldsymbol{v}_\alpha \varphi_\alpha\rangle\right] - n_\alpha\left[\left\langle \boldsymbol{v}_\alpha \frac{\partial \varphi_\alpha}{\partial \boldsymbol{x}}\right\rangle + \boldsymbol{F}_\alpha \left\langle \frac{\partial \varphi_\alpha}{\partial \boldsymbol{p}_\alpha}\right\rangle\right] = \Delta_\alpha \tag{8}$$

for the functions $\varphi_\alpha\,(\boldsymbol{x}, \boldsymbol{p}_\alpha, \varepsilon_\alpha)$.

$$\langle \varphi_\alpha \rangle = (1/n_\alpha) \int \varphi_\alpha f_\alpha \,(\mathrm{d}^3 p_\alpha/h^3) \tag{9}$$

denote mean values of φ_α, where n_α is the particle density and h signifies Planck's constant. The normalization of the distribution functions follows from substitution of $\varphi_\alpha = \langle \varphi_\alpha \rangle = 1$. The quantity

$$\Delta_\alpha = \int \varphi_\alpha \frac{\partial_s f_\alpha}{\partial t}\frac{d^3 p_\alpha}{h^3} \tag{10}$$

describes the collisional rate of change of φ_α. As far as Δ_α takes into account interactions between material particles and the radiation field, the particle moment equations must be supplemented by the radiation transport equation, which in detail has been investigated by SAMPSON [10].

To justify a truncation of the set of moment equations and to evaluate the collision integrals we shall use the particle distribution functions in the near equilibrium approximation

$$f_\alpha = n_\alpha [h^2 \beta_\alpha/\pi m_\alpha^2]^{3/2} \exp[-\beta_\alpha p_\alpha^2/m_\alpha^2]\,(1+\Psi_\alpha), \tag{11}$$

where $\psi_\alpha \ll 1$. The plasma is assumed to be at rest.

In the 8-moment approximation of Grad's polynomial expansion ψ_α is given by the expression

$$\psi_\alpha = 2\beta_\alpha \boldsymbol{w}_\alpha \frac{\boldsymbol{p}_\alpha}{m_\alpha} + \frac{4}{5}\beta_\alpha \frac{\boldsymbol{h}_\alpha}{P_\alpha}\frac{\boldsymbol{p}_\alpha}{m_\alpha}\left[\frac{\beta_\alpha p_\alpha^2}{m_\alpha^2} - \frac{5}{2}\right], \tag{12}$$

which generates the following system of balance equations for

particle density $\varphi_\alpha = 1, \quad (\partial/\partial \boldsymbol{x})\,(n_\alpha \boldsymbol{w}_\alpha) = G_\alpha,$ (13)

momentum $\varphi_\alpha = \boldsymbol{p}_\alpha, \quad (\partial P_\alpha/\partial \boldsymbol{x}) - n_\alpha e_\alpha \boldsymbol{E} = \boldsymbol{I}_\alpha,$ (14)

total energy $\varphi_\alpha = \varepsilon_\alpha + (p_\alpha^2/2m_\alpha)$,

$$(\partial \boldsymbol{q}_\alpha/\partial \boldsymbol{x}) + \varepsilon_\alpha G_\alpha - n_\alpha e_\alpha \boldsymbol{w}_\alpha \boldsymbol{E} = Q_\alpha,$$

heat flux

$$\varphi_\alpha = \frac{\boldsymbol{p}_\alpha}{2\beta_\alpha}\left[\frac{\beta_\alpha p_\alpha^2}{m_\alpha^2} - \frac{5}{2}\right], \quad c_{P\alpha} P_\alpha\,(\partial T_\alpha/\partial \boldsymbol{x}) = \boldsymbol{S}_\alpha \tag{15}$$

where $c_{P\alpha} = 5k/2m_\alpha$ is the specific heat per unit mass at constant static pressure $P_\alpha = n_\alpha k T_\alpha$. The quantities

$$\boldsymbol{w}_\alpha = \frac{1}{n_\alpha}\int \frac{\boldsymbol{p}_\alpha}{m_\alpha} f_\alpha \frac{\mathrm{d}^3 p_\alpha}{h^3}, \quad \boldsymbol{q}_\alpha = \int \frac{p_\alpha^2 \boldsymbol{p}_\alpha}{2m_\alpha^2} f_\alpha \frac{\mathrm{d}^3 p_\alpha}{h^3} \tag{16}$$

characterize diffusion velocity and energy flux of the component α. G_α, $\boldsymbol{I}_\alpha$, Q_α, and $\boldsymbol{S}_\alpha$ signify respectively the collisional rates of change of particle density, momentum, total energy, and heat flux. Heat and energy flux are coupled by the relation

$$\boldsymbol{h}_\alpha = \boldsymbol{q}_\alpha - \tfrac{5}{2} P_\alpha \boldsymbol{w}_\alpha. \tag{17}$$

IV. Formulation of the Collision Integrals

a) In case of the binary elastic, inelastic, and superelastic collision process

$$A_i + \bar{A}_k \rightleftharpoons A_j' + \bar{A}_l'$$

the collision operator for the component A_i has the usual Boltzmann form

$$\partial_s f_i/\partial t = \sum_{jkl} \int [\Lambda f_j' \bar{f}_l' - f_i \bar{f}_k]\; g_{ik}\, I\overset{jl'}{\underset{ik}{\uparrow}}\, \mathrm{d}\Omega'\,(\mathrm{d}^3 \bar{p}_k/h^3) \tag{18}$$

where

$$\Lambda \equiv g_i \bar{g}_k / g_j \bar{g}_l \tag{19}$$

[10] D. H. SAMPSON, Radiative Contributions to Energy and Momentum Transport in a Gas, Interscience Publishers 1965.

is the ratio of the statistical weights of the different internal energy states $\varepsilon_i \ldots \bar{\varepsilon}_l$. and

$$\mathrm{d}\Omega' = \sin\chi \,\mathrm{d}\chi \,\mathrm{d}\eta \tag{20}$$

signifies the solid angle element for the scattering process in the center-of-mass system of the collision partners. The direction of the collision process is indicated by the vertical arrow in the differential collision cross section $I \underset{ik}{\overset{jl'}{\uparrow}} (\Delta_{ikjl}, g_{ik}, \Omega')$. The latter is related to the quantum mechanical transition probability $\overline{|W \underset{ik}{\overset{jl'}{\uparrow}} (g_{ik}, g'_{jl}, \Omega')|^2}$ summed over all degenerate final states and averaged over all degenerate initial states

$$\overline{\left| W \underset{ik}{\overset{jl'}{\uparrow}} \right|^2} = \frac{1}{g_i \bar{g}_k} \sum_{ijkl}^{\text{deg.st.}} \left| W \underset{ik}{\overset{jl'}{\uparrow}} \right|^2 \tag{21}$$

through the relation

$$I \underset{ik}{\overset{jl'}{\uparrow}} = \int \overline{\left| W \underset{ik}{\overset{jl'}{\uparrow}} \right|^2} \delta\,(g'^2_{jl} - R^2 g^2_{ik}) \frac{g'^2_{jl}}{g_{ik}} \mu^2 \frac{\mathrm{d}g'^2_{jl}}{h^6}. \tag{22}$$

The Boltzmann collision operator used is based on the microscopic reversibility relation

$$\overline{\left| W \underset{ik}{\overset{jl'}{\downarrow}} \right|^2} = \Lambda \overline{\left| W \underset{ik}{\overset{jl'}{\uparrow}} \right|^2}, \tag{23}$$

which in terms of the differential collision cross sections reads

$$g'^2_{jl}\, I \underset{ik}{\overset{jl'}{\downarrow}} = \Lambda\, g^2_{ik}\, I \underset{ik}{\overset{jl'}{\uparrow}}, \tag{24}$$

and the Liouville transformation [4]

$$g_{ik}\,\mathrm{d}\Omega\,\mathrm{d}^3p_j'\,\mathrm{d}^3\bar{p}_l' = g'_{jl}\,\mathrm{d}\Omega'\,\mathrm{d}^3p_i\,\mathrm{d}^3\bar{p}_k. \tag{25}$$

With Eqs. (24), (25) interchanging primed and unprimed quantities we find from Eq. (18)

$$\Delta_i = \sum_{jklm} \int [\varphi_j' \delta_{ij} - \varphi_m \delta_{im}] \cdot f_m \bar{f}_k g_{mk} I \underset{mk}{\overset{jl'}{\uparrow}} \mathrm{d}\Omega' (\mathrm{d}^3 p_m \mathrm{d}^3 \bar{p}_k / h^6) \tag{26}$$

where δ_{ij} and δ_{im} are Kronecker symbols working on the internal energy quantum numbers. The collision integrals Δ_i may be summed over all internal states i to give the simplified expression

$$\sum_i \Delta_i = \sum_{ijkl} \int [\varphi_j' - \varphi_i] \cdot f_i \bar{f}_k g_{ik} I \underset{ik}{\overset{jl'}{\uparrow}} \mathrm{d}\Omega' (\mathrm{d}^3 p_i \mathrm{d}^3 \bar{p}_k / h^6) \tag{27}$$

An analogous equation holds for $\sum_k \bar{\Delta}_k$ replacing $\varphi_j' - \varphi_i$ by $\bar{\varphi}' - \bar{\varphi}_k$ so that we have $\sum_i \Delta_i + \sum_k \bar{\Delta}_k = 0$ for the summational invariants.

b) In case of the photo-ionization and radiative recombination process

$$A_i + h\nu \rightleftharpoons A_+ + \mathrm{e}_-$$

the collision operator for the component A_i may be written in the form

$$\frac{\partial_s f_i}{\partial t} = \int g_{-+} \sigma \underset{\nu i}{\overset{-+}{\downarrow}} f_- f_+ \left.\frac{\mathrm{d}^3 p_{-+}}{h^3}\right|_{P_{-+}=p_i} - \int c\, \sigma \underset{\nu i}{\overset{-+}{\uparrow}} f_i f_\nu 2 \frac{\mathrm{d}^3 p_\nu}{h^3} \tag{28}$$

where the ion species due to its high excitation energies is supposed to exist in its internal energy ground state only.

In writing Eq. (28) the momentum coordinates of the electrons (index $-$) and ions (index $+$) have been transformed to relative and center-of-mass momenta $\boldsymbol{p}_{-+} = \mu_{-+} \boldsymbol{g}_{-+}$ and $\boldsymbol{P}_{-+} = \boldsymbol{p}_- + \boldsymbol{p}_+$. Moreover, due to momentum conservation the integration over $\boldsymbol{P}_{-+}$ has already been performed neglecting the photon momentum $p_\nu = h\nu/c$ in comparison to the particle momentum difference $|\boldsymbol{p}_i - \boldsymbol{P}_{-+}|$. f_ν is the distribution function of the photons, which are assumed to have random polarization. The degeneracy concerning the polarization state of the photons is taken into account by the factor 2. The total cross sections for radiative recombination $\sigma \underset{\nu i}{\overset{-+}{\downarrow}} (g_{-+}, \varepsilon_+ - \varepsilon_i)$ and photo-ionization $\sigma \underset{\nu i}{\overset{-+}{\uparrow}} (\nu, \varepsilon_+ - \varepsilon_i)$ are defined by the integrals

$$\sigma \underset{\nu i}{\overset{-+}{\downarrow}} = \frac{1}{g_{+-}} \int \frac{\overline{\left| W \underset{\nu i}{\overset{-+}{\downarrow}} \right|^2}}{h^3} (1 + f_\nu) \cdot \delta \left[\frac{p^2_{-+}}{2\mu_{-+}} + \varepsilon_+ - \varepsilon_i - h\nu \right] 2 \frac{\mathrm{d}^3 p_\nu}{h^3} \tag{29}$$

and

$$\sigma \underset{\nu i}{\overset{-+}{\uparrow}} = \frac{1}{c} \int \frac{\overline{\left| W \underset{\nu i}{\overset{-+}{\uparrow}} \right|^2}}{h^3} \delta \left[\frac{p^2_{-+}}{2\mu_{-+}} + \varepsilon_+ - \varepsilon_i - h\nu \right] \frac{\mathrm{d}^3 p_{-+}}{h^3}. \tag{30}$$

In contrast to the usual Boltzmann operator the collision term for radiative recombination contains the factor $1 + f_\nu$, which describes the combined effect of spontaneous and induced recombination photon emission. Line broadening effects are neglected. The line shape is consequently given by a Delta-function in energy space. The microscopic reversibility of the radiative ionization-recombination process can be written in the form

$$\overline{\left| W \overset{-+}{\underset{\nu i}{\uparrow}} \right|^2} = \frac{g_- g_+}{g_i} \overline{\left| W \overset{-+}{\underset{\nu i}{\downarrow}} \right|^2} \tag{31}$$

where the averaged transition probabilities are defined by the relations

$$\overline{\left| W \overset{-+}{\underset{\nu i}{\uparrow}} \right|^2} = \frac{1}{2 g_i} \sum^{\text{deg.st.}} \left| W \overset{-+}{\underset{\nu i}{\uparrow}} \right|^2,$$

$$\overline{\left| W \overset{-+}{\underset{\nu i}{\downarrow}} \right|^2} = \frac{1}{2 g_- g_+} \sum^{\text{deg.st.}} \left| W \overset{-+}{\underset{\nu i}{\downarrow}} \right|^2 \tag{32}$$

the summation being extended over all degenerate particle states and all directions of photon polarization. The factor $g_- = 2$ accounts for the two spin directions of the electrons.

From Eq. (28) we obtain the collision integral

$$\Lambda_i = \int g_{-+}\, \sigma \overset{-+}{\underset{\nu i}{\downarrow}} \varphi_i f_- f_+ \left| \frac{\mathrm{d}^3 p_i\, \mathrm{d}^3 p_{-+}}{h^6} \right|_{P_{-+} = p_i} - \beta \overset{-+}{\underset{\nu i}{\uparrow}} n_i \langle \varphi_i \rangle \tag{33}$$

where

$$\beta \overset{-+}{\underset{\nu i}{\uparrow}} = \int c\, \sigma \overset{-+}{\underset{\nu i}{\uparrow}} f_\nu\, 2\, (\mathrm{d}^3 p_\nu / h^3) \tag{34}$$

is the rate coefficient for photo-ionization.

We note that by means of the microscopic reversibility, Eq. (31), and the Planck photon distribution the equilibrium relation[11]

$$\sigma \overset{-+}{\underset{\nu i}{\downarrow}} = \frac{(g_i/g_+)\, (h \nu / c\, p_{-+})^2}{1 - \exp(-h \nu / k T)}\, \sigma \overset{-+}{\underset{\nu i}{\uparrow}} \tag{35}$$

follows from the definition of the photo-ionization and radiative recombination cross sections.

In the dipole approximation the transition probability and the total cross section for photo-ionization, when averaged over the solid angle Ω_ν with an isotropic f_ν, are given by the expressions (in MKSA-units)

$$\int \frac{\overline{\left| W \overset{-+}{\underset{\nu i}{\uparrow}} \right|^2}}{h^3} \frac{\mathrm{d}\, \Omega_\nu}{4 \pi} = \frac{\left[\dfrac{p_{-+}^2}{2 \mu_{-+}} - \varepsilon_i + \varepsilon_+ \right]^2}{6 \varepsilon_0 \hbar^2 \nu} \cdot \frac{1}{g_i} \sum_{i-+}^{\text{deg.st.}} \left| e\, \boldsymbol{r}_{\boldsymbol{p}_{-+} i} \right|^2 (2 \pi \hbar)^3, \tag{36}$$

$$\sigma \overset{-+}{\underset{\nu i}{\uparrow}} = \frac{e^2 \nu\, \mu_{-+} p_{-+}}{3 \varepsilon_0 c\, \hbar^3} \frac{1}{g_i} \sum_{i-+}^{\text{deg.st.}} \left| \boldsymbol{r}_{\boldsymbol{p}_{-+} i} \right|^2 (2 \pi \hbar)^3 \tag{37}$$

provided that in the transition matrix $\boldsymbol{r}_{\boldsymbol{p}_{-+} i}$ the wave function of the continuum state of the electron-ion pair is normalized to the Delta-function of the relative momentum $\boldsymbol{p}_{-+}$.

c) In case of the radiative excitation and deexcitation of atomic energy levels

$$A_i \rightleftharpoons A_j + h \nu$$

the collision operator for the component A_i regarding transitions from and to lower energy levels $(j < i)$ takes the form

$$\frac{\partial_s f_i}{\partial t} = \sum_{j<i} \int \delta (h \nu - \varepsilon_i + \varepsilon_j) \frac{\overline{\left| W \overset{i}{\underset{\nu j}{\downarrow}} \right|^2}}{h^3} \cdot \left[\Lambda f_j f_\nu \big|_{p_j = p_i} - f_i (1 + f_\nu) \right] 2 \frac{\mathrm{d}^3 p_\nu}{h^3} \tag{38}$$

where we again assume negligably small momentum as well as random polarization of the photons and neglect line broadening effects, which should replace the energy Delta-function by a finit line shape factor. Use has been made of the microscopic reversibility relation

$$\overline{\left| W \overset{i}{\underset{\nu j}{\uparrow}} \right|^2} = \Lambda \overline{\left| W \overset{i}{\underset{\nu j}{\downarrow}} \right|^2} \quad \text{with} \quad \Lambda = \frac{g_i}{g_j}$$

and

$$\overline{\left| W \overset{i}{\underset{\nu j}{\downarrow}} \right|^2} = \frac{1}{2 g_i} \sum_{\nu i j}^{\text{deg.st.}} \left| W \overset{i}{\underset{\nu j}{\downarrow}} \right|^2. \tag{39}$$

In the rest frame of A_i resp. A_j assuming an isotropic photon distribution the dipole approximation yields the well known formula for the transition probability averaged over the solid angle Ω_ν

$$\int \frac{\overline{\left| W \overset{i}{\underset{\nu j}{\downarrow}} \right|^2}}{h^3} \frac{\mathrm{d}\, \Omega_\nu}{4 \pi} = \frac{e^2 (\varepsilon_i - \varepsilon_j)^2}{6 \varepsilon_0 \hbar^2 \nu} \frac{1}{g_i} \sum_{ij}^{\text{deg.st.}} \left| \boldsymbol{r}_{ji} \right|^2. \tag{40}$$

The collision operator can be reduced to a simple form including the Einstein coefficients for spontaneous and induced emission of radiation, which we define by the integrals

$$A \overset{i}{\underset{\bar{\nu} j}{\downarrow}} = \int \delta (h \nu - \varepsilon_i + \varepsilon_j) \frac{\overline{\left| W \overset{i}{\underset{\nu j}{\downarrow}} \right|^2}}{h^3} 2 \frac{\mathrm{d}^3 p_\nu}{h^3} \tag{41}$$

$$B \overset{i}{\underset{\bar{\nu} j}{\downarrow}} u_\nu = \int \delta (h \nu - \varepsilon_i + \varepsilon_j) \frac{\overline{\left| W \overset{i}{\underset{\nu j}{\downarrow}} \right|^2}}{h^3} f_\nu\, 2 \frac{\mathrm{d}^3 p}{h^3}, \quad \cdot B \overset{i}{\underset{\bar{\nu} j}{\uparrow}} = \Lambda B \overset{i}{\underset{\bar{\nu} j}{\uparrow}} \tag{42}$$

[11] D. W. Norcross and P. M. Stone, J. Quant. Spectrosc. Radiat. Transfer 6, 277 [1966].

where $B \overset{i}{\underset{\bar{\nu}j}{\uparrow}}$ is the Einstein coefficient for photon absorption and

$$u_{\bar{\nu}} = (8\pi/c^3)\, h\,\bar{\nu}^3 f_{\bar{\nu}} \quad \text{with} \quad h\,\bar{\nu} \equiv \varepsilon_i - \varepsilon_j \tag{43}$$

is the radiation energy density per unit frequency for isotropic f_ν.

The relativistic formulae[12]

$$\frac{\delta(h\,\nu - \varepsilon_i + \varepsilon_j)}{1-(\boldsymbol{p}_i\,\boldsymbol{s}/m\,c)} \left| W \overset{i}{\underset{\nu j}{\downarrow}} \right|^2, \nu\,\mathrm{d}\nu\,\mathrm{d}\Omega_\nu = \text{inv.} \quad [\boldsymbol{s} = \boldsymbol{p}_\nu/p_\nu], \tag{44}$$

show in connection with the Doppler formula that in the limit $(p_i/mc)^2 \ll 1$ the photon integrals for the Einstein coefficients are independent of the coordinate frame. When evaluated by means of Eq. (40) the usual expressions for the Einstein coefficients in the dipole approximation are recovered.

Taking the velocity moment of the collision operator we have from Eq. (38)

$$\begin{aligned} \Delta_i = \sum_{j<i} \Bigg[& B \overset{i}{\underset{\bar{\nu}j}{\uparrow}} u_{\bar{\nu}} \int \varphi_i \left| f_j \frac{\mathrm{d}^3 p_j}{h^3} \right|_{\boldsymbol{p}_i = \boldsymbol{p}_j} \\ & - (A \overset{i}{\underset{\bar{\nu}j}{\downarrow}} + B \overset{i}{\underset{\bar{\nu}j}{\downarrow}} u_{\bar{\nu}})\, n_i \langle \varphi_i \rangle \Bigg]. \end{aligned} \tag{45}$$

An analogous equation holds for $j > i$. Considering all values of j we can prove that $\sum_i \Delta_i = 0$, if φ_i depends on the momentum $\boldsymbol{p}_i$ only. This is an obvious consequence of the fact that

$$h\,\nu/c \ll |\boldsymbol{p}_i - \boldsymbol{p}_j| \,.$$

V. Evaluation of the Collision Integrals for Binary Particle Interactions

Changing to modified relative and center-of-mass velocities $\boldsymbol{\zeta}_{ik}$ and $\boldsymbol{V}_{ik}$, Eq. (5), and making the linear approximation

$$\begin{aligned} f_m \bar{f}_k \cong & \frac{n_m \bar{n}_k}{\pi^3} \left| \frac{h^2 \sqrt{\beta\,\bar{\beta}}}{m\,\bar{m}} \right|^3 \exp(-V_{mk}^2 - \zeta_{mk}^2) \\ & \cdot (1 + \psi_m + \bar{\psi}_k) \end{aligned} \tag{46}$$

the Boltzmann collision integrals for binary elastic, inelastic, and superelastic particle collisions, Eq.(26), are evaluated within Grad's 8-moment approximation for ψ_m and $\bar{\psi}_k$. The evaluation is simplified by the assumption of axially symmetric differential collision cross sections

$$\partial I \overset{jl'}{\underset{mk}{\uparrow}} / \partial\eta = 0\,. \tag{47}$$

This assumption leads to integrals of the form

$$\int_0^{2\pi} [\varphi_j' \delta_{ij} - \varphi_m \delta_{im}]\,\mathrm{d}\eta \quad \text{resp.} \quad \int_0^{2\pi} [\varphi_j' - \varphi_i]\,\mathrm{d}\eta \tag{48}$$

which easily can be performed. Moreover, the fact that the collision cross sections only depend on magnitude and direction of the relative velocities of the collision partners enables a direct integration over the modified center-of-mass velocity $\boldsymbol{V}_{mk}$ changing the collision integrals into linear functions of generalized Chapman-Cowling integrals

$$\Omega_{mk\to jl}^{(\lambda\varrho)} = \sqrt{\frac{\pi}{\gamma}} \int_0^\infty \zeta^{2\varrho+3} \exp(-\zeta^2)\, \varphi_{mk\to jl}^{(\lambda)}(\zeta, \Delta)\,\mathrm{d}\zeta \tag{49}$$

where the identities $\zeta \equiv \zeta_{mk}$ and $\Delta \equiv \Delta_{mkjl}$ hold. We realize that

$$\varphi_{mk\to jl}^{(\lambda)} = \int_0^\pi [1 - R^\lambda \cos^\lambda \chi]\, I \overset{jl'}{\underset{mk}{\uparrow}} (\zeta, \Delta, \chi) \sin\chi\,\mathrm{d}\chi \tag{50}$$

are the gaskinetic collision cross sections divided by 2π, noticing that for $\lambda = -\infty$ Eq. (50) defines total collision cross sections.

Performing the integrations the following expressions can be derived for the collision integrals:

$$G_i = 8 \sum_{jklm} (\delta_{ij} - \delta_{im})\, n_m \bar{n}_k\, \Omega^{(-\infty 0)}\,, \tag{51}$$

$$Q_i|_{\text{int.}} = \varepsilon_i G_i\,, \tag{52}$$

$$Q_i|_{\text{tr.}} = 8 \sum_{jklm} \delta_{ij} n_m \bar{n}_k (\mu/\gamma)\, c_1 [\Omega^{(\lambda\varrho)}] + \tfrac{3}{2} k\,T [G_i - 8 \sum_{jklm} (\delta_{ij} - \delta_{im})\, n_m \bar{n}_k\, \bar{\Gamma}\, \overline{\omega}^{(-\infty 0)}]\,, \tag{53}$$

$$\begin{aligned} \boldsymbol{I}_i = & \tfrac{16}{3} \sum_{jklm} \delta_{ij} n_m \bar{n}_k \mu [(\bar{\boldsymbol{w}}_k - \boldsymbol{w}_m)\, \Omega^{(11)} - 2\gamma (\bar{\boldsymbol{H}}_k - \boldsymbol{H}_m)\, \omega^{(11)}] \\ & + \tfrac{16}{3} \sum_{jklm} (\delta_{ij} - \delta_{im})\, n_m \bar{n}_k m [(\bar{\Gamma} \bar{\boldsymbol{w}}_k + \Gamma \boldsymbol{w}_m)\, \tfrac{3}{2}\, \Omega^{(-\infty 0)} - (\bar{\boldsymbol{w}}_k - \boldsymbol{w}_m)\, \bar{\Gamma}\, \Omega^{(-\infty 1)} \\ & - 2\gamma (\bar{\Gamma} \bar{\boldsymbol{H}}_k + \Gamma \boldsymbol{H}_m)^3\, \overline{\omega}^{(-\infty 0)} + 2\gamma (\bar{\boldsymbol{H}}_k - \boldsymbol{H}_m)\, \bar{\Gamma}\, \omega^{(-\infty 1)}], \end{aligned} \tag{54}$$

[12] L. D. Landau and E. M. Lifshitz, The Classical Theory of Fields, Pergamon Press, London 1962.

$$\begin{aligned}\boldsymbol{S}_i = & \tfrac{8}{3} \sum_{jklm} \delta_{ij}\, n_m \bar{n}_k\, (\mu/\gamma)\, \{(\bar{\Gamma} \bar{\boldsymbol{w}}_k + \Gamma \boldsymbol{w}_m)\, 5\, c_1[\Omega^{(\lambda\varrho)}] - (\bar{\boldsymbol{w}}_k - \boldsymbol{w}_m)\, c_2[\Omega^{(\lambda\varrho)}] \\ & - 2\gamma (\bar{\Gamma} \bar{\boldsymbol{H}}_k + \Gamma \boldsymbol{H}_m)\, (5\, c_1[\omega^{(\lambda\varrho)}] - \tfrac{4}{5} c_3[\Omega^{(\lambda\varrho)}]) + 2\gamma (\bar{\boldsymbol{H}}_k - \boldsymbol{H}_m)\, c_2[\omega^{(\lambda\varrho)}] \\ & + 2\gamma (\bar{\Gamma}^2 \bar{\boldsymbol{H}}_k - \Gamma^2 \boldsymbol{H}_m)\, 3\, \Omega^{(11)}\} + \tfrac{16}{3} \sum_{jklm} (\delta_{ij} - \delta_{im})\, n_m \bar{n}_k\, k\, T\, [\ldots]\end{aligned} \tag{55}$$

where the dotted bracket contains a rather lengthy expression in diffusion velocity and heat flux.

The coefficients $c_1 .. c_3$ have the following meaning:

$$c_1[\Omega^{(\lambda\varrho)}] = (\bar{M} - \bar{\Gamma})\, \Omega^{(11)} + \frac{\gamma \Delta}{\mu} \bar{M}\, \Omega^{(-\infty 0)}, \tag{56}$$

$$c_2[\Omega^{(\lambda\varrho)}] = \left(\frac{5}{2} \bar{\Gamma}^2 - \frac{2\gamma\Delta}{\mu} \bar{M}^2\right) \Omega^{(11)} - (3\, (\bar{M} - \bar{\Gamma})^2 - \bar{M}^2)\, \Omega^{(12)} + 2\, \bar{M}\, (\bar{M} - \bar{\Gamma})\, \Omega^{(22)} + \frac{2\gamma\Delta}{\mu} \bar{M}\, \bar{\Gamma}\, \Omega^{(-\infty 1)}, \tag{57}$$

$$c_3[\Omega^{(\lambda\varrho)}] = 3\, (\bar{M} - \bar{\Gamma})\, \Omega^{(12)} - \bar{M}\, \Omega^{(22)} + \frac{\gamma\Delta}{\mu} \bar{M}\, \Omega^{(-\infty 1)}. \tag{58}$$

Moreover the abbreviations

$$\boldsymbol{H}_i \equiv (\boldsymbol{h}_i / \varrho_i)\,, \quad \bar{\omega}^{(-\infty 0)} \equiv \Omega^{(-\infty 0)} - \tfrac{2}{3}\, \Omega^{(-\infty 1)}\,, \quad \omega^{(\lambda\varrho)} \equiv \Omega^{(\lambda\varrho)} - \tfrac{2}{5}\, \Omega^{(\lambda\varrho+1)}, \quad \varrho_i \equiv n_i m_i \tag{59}$$

have been introduced.

Because of the recurrence relations

$$\Omega^{(\lambda\varrho+1)} = (\varrho + \tfrac{3}{2})\, \Omega^{(\lambda\varrho)} - \frac{\mathrm{d}\Omega^{(\lambda\varrho)}}{\mathrm{d}(\ln \gamma)} \tag{60}$$

the transport coefficients can be equivalently expressed either through the first few moments of the collision cross sections or through their derivatives.

We notice that in order to simplify the notation the subindices $mk \to jl$ of the Chapman-Cowling integrals are omitted. To preserve the order of subindices $mk \to jl$ in all Chapman-Cowling integrals appearing in the above formulae the Kronecker symbols δ_{im} and δ_{ij} have been retained. It is easily seen from inspection of the sums over the internal energy state numbers that terms proportional to δ_{im} are due to processes $ik \to jl$, whereas terms proportional to δ_{ij} characterize the inverse processes $jl \to ik$. Note that the Chapman-Cowling integrals for the processes $jl \to mk$ involving total cross sections are given by the relation

$$\sqrt{\frac{\pi}{\gamma}} \left(-\frac{2\gamma\Delta}{\mu}\right) \Omega^{(-\infty\varrho)}_{jl \to mk} = \Delta \int_0^\infty \left[1 + \frac{2\gamma\Delta}{\mu\,\zeta^2}\right]^\varrho \cdot \zeta^{2\varrho+3} \exp(-\zeta^2)\, \varphi^{(-\infty)}_{mk \to jl}(\zeta, \Delta)\, \mathrm{d}\zeta. \tag{61}$$

Making use of this relation we see that $G_i = 0$ yields the correct Boltzmann distribution over the internal energy states in thermal equilibrium.

Considering only terms proportional to δ_{im} binary processes such as radiative recombination and ionization by electron impact inverting photon and three body processes are also included within the above formalism as can be seen from Eq. (26). Of course, the Chapman-Cowling integrals for such processes only involve total cross sections. In case of radiative recombination, for instance, we define the integrals

$$\Omega^{(\varrho)}_{-+\to\nu i} = \sqrt{\frac{\pi}{\gamma_{-+}}} \int_0^\infty \zeta^{2\varrho+3} \exp(-\zeta^2)\, \frac{\overset{-+}{\underset{\nu i}{\sigma\downarrow}}(\zeta, \varepsilon_+ - \varepsilon_i)}{2\pi}\, \mathrm{d}\zeta \tag{62}$$

where we have $\zeta \equiv \zeta_{-+}$.

As indicated by Eq. (27) the collision integrals are considerably simplified, if the contributions of all internal energy states of the same particle species are added. In fact, all terms proportional to $\delta_{ij} - \delta_{im}$ vanish when summed over the internal energy index i.

VI. Simplifications of the Collision Integrals

Within Grad's 8-moment approximation of the distribution functions neglecting bilinear terms $\psi_m \bar{\psi}_k$ Eq. (51) to (55) are quite general and hold for all components involved in binary collisions between material particles in monatomic plasmas. Since in their general form the collision integrals are hardly needed for practical cases, it is important to note characteristic simplifications that result from specification of the components A_i and $\bar{A}_k$.

atom-atom-interactions: If $\bar{A} \neq A$, we are dealing with a Penning mixture. Since both A_i and $\bar{A}_k$ are heavy particle components, we may assume thermal equilibrium between the translational energy states ($\bar{T} = T$).

In addition, if $\bar{A}_k = A_k$, further simplifications of the collision integrals are possible on account of the mass equality of the collision partners. Note that in our approximation elastic interactions between particles of the same component contribute only to the heat flux

$$S_i = -\tfrac{16}{15}\, n_i\, \Omega_i^{(22)}\, \varrho_i\, \boldsymbol{H}_i\,. \tag{63}$$

atom-ion-interactions ($\bar{A}_k = A_+$): From the reasons given above we may again assume thermal equilibrium between the translational degrees of freedom ($T_+ = T$). In a low temperature monatomic plasma excitation collisions of the ions are unlikely to occur and therefore we need only consider elastic collisions (including charge exchange). For these we find from the general equations:

$$G_i = 0; \quad \boldsymbol{I}_i = \tfrac{1}{2}\, \varrho_i\, \nu_{i+}\left[(\boldsymbol{w}_+ - \boldsymbol{w}_i) - \beta(\boldsymbol{H}_+ - \boldsymbol{H}_i)\, \frac{\omega_{i+}^{(11)}}{\Omega_{i+}^{(11)}}\right]; \quad Q_i = 0\,, \tag{64}$$

$$\boldsymbol{S}_i = \tfrac{1}{4}\, P_i\, \nu_{i+}\left[-(\boldsymbol{w}_+ - \boldsymbol{w}_i)\, \frac{5}{2}\, \frac{\omega_{i+}^{(11)}}{\Omega_{i+}^{(11)}} - \frac{4}{5}\, \beta(\boldsymbol{H}_+ + \boldsymbol{H}_i)\, \frac{\Omega_{i+}^{(22)}}{\Omega_{i+}^{(11)}} + \beta(\boldsymbol{H}_+ - \boldsymbol{H}_i)\left(\frac{\tfrac{5}{2}\,\omega_{i+}^{(11)} - \omega_{i+}^{(12)}}{\Omega_{i+}^{(11)}} + 3\right)\right] \tag{65}$$

with the average momentum transfer collision frequency

$$\nu_{i+} \equiv \tfrac{16}{3}\, n_+\, \Omega_{i+}^{(11)}\,. \tag{66}$$

ion-atom interactions: For all conserved quantities the collision integrals of the ions are related to those of the neutrals, Eq. (64), by $\Delta_+ = -\sum_i \Delta_i$. For $\boldsymbol{S}_i$ we get an expression, which differs from $\sum_i$ of Eq. (65) by the sign of diffusion and heat flux differences.

atom-electrons interactions ($\bar{A}_k = \mathrm{e}_-$): In this case $T_- > T$ is possible for reasons well known. The smallness of m_-/m and the lack of internal electron structure ($\varepsilon_k = \varepsilon_l = 0$) lead to considerable simplifications of the collision integrals. In particular, compared with heavy particle collisions, electron collisions have only a small effect on the collisional rate of momentum and heat transfer of the atoms. Electron collisions can, however, greatly affect the production and loss rates of number density and total energy of the excited atom states. For these we have

$$G_i = 8\, n_- \sum_j \left[n_j\, \Omega_{j\to i,-}^{(-\infty 0)} - n_i\, \Omega_{i\to j,-}^{(-\infty 0)}\right], \tag{67}$$

$$Q_i \cong \frac{m_-}{m}\, 3\, k\, T_-\left(1 - \frac{T}{T_-}\right) \sum_j n_j\, \nu_{j\to i,-} + \left(\tfrac{3}{2}\, k\, T + \varepsilon_i\right) G_i \tag{68}$$

with

$$\nu_{j\to i,-} \equiv \tfrac{16}{3}\, n_-\, \Omega_{j\to i,-}^{(11)} \tag{69}$$

in analogy to Eq. (66). Eq. (67), (68) apply to exciting and deexciting electron collisions. Ionizing electron collisions may be incorporated through the expression

$$G_i = -\, 8\, n_-\, n_i\, \Omega_{i\to +,-}^{(-\infty 0)}\,. \tag{70}$$

electron-atom interactions: For all conserved quantities we again have $\Delta_- = -\sum_i \Delta_i$. Note, however, that in the electron collision integrals for momentum and heat transfer heavy particle collisions are not necessarily a small effect. Making use of the inequalities $|\boldsymbol{w}_-| \gg |\boldsymbol{w}_i|$ and $|\boldsymbol{h}_-/P_-| \gg |\boldsymbol{h}_i/P_i|$, which hold for many applications, we obtain the following simplified set of electron collision integrals

$$G_- = 0; \quad \boldsymbol{I}_- \cong \sum_{ij} \varrho_-\, \nu_{-,j\to i}\left[-\boldsymbol{w}_- + 2\,\beta_-\, \boldsymbol{H}_-\, \frac{\omega_{j\to i}^{(11)}}{\Omega_{j\to i}^{(12)}}\right], \tag{71}$$

$$Q_- = -\frac{m_-}{m}\, 3\, k\, T_-\left(1 - \frac{T}{T_-}\right) \sum_{ij} n_-\, \nu_{-,j\to i} + 8\, n_- \sum_{ij} n_j\, (\varepsilon_j - \varepsilon_i)\, \Omega_{j\to i,-}^{(-\infty 0)}\,, \tag{72}$$

$$\boldsymbol{S}_- \cong P_- \sum_{ij} \nu_{-,j\to i}\left[\boldsymbol{w}_- \frac{5}{2}\, \frac{\omega_{j\to i,-}^{(11)}}{\Omega_{j\to i,-}^{(11)}} - 2\,\beta_-\, \boldsymbol{H}_-\, \frac{\tfrac{5}{2}\,\omega_{j\to i,-}^{(11)} - \omega_{j\to i,-}^{(12)}}{\Omega_{j\to i,-}^{(11)}}\right] \tag{73}$$

with

$$\nu_{-,j\to i} \equiv \tfrac{16}{3}\, n_j\, \Omega_{j\to i,-}^{(11)}\,. \tag{74}$$

In the above formulae only exciting and deexciting electron collisions are considered so that we have $G_- = 0$ as an obvious result. To include contributions from ionizing electron collisions (without consideration of the inverse three body collisions) we must add to Eq. (71) to (73) expressions, which can be derived from terms proportional to δ_{im} in the general set of Eq. (51) to (55).

Assuming that on the whole the electrons lose nearly their total energy during an inelastic collision we may also simplify the corresponding Chapman-Cowling integrals through the approximation

$$\Omega^{(\lambda\varrho)}_{j\to i,-} \cong \Omega^{(-\infty\varrho)}_{j\to i,-} \quad \text{for} \quad i \neq j \tag{75}$$

which produces

$$\sum_{ij} n_j \, \Omega^{(1\varrho)}_{j\to i,-} \cong \sum_{i} n_i \, \Omega^{(1\varrho)}_{i-} + \sum_{\substack{ij \\ i \neq j}} n_j \, \Omega^{(-\infty\varrho)}_{j\to i,-} \,. \tag{76}$$

In this approximation inelastic electron collisions are described by total collision cross sections, which are available from GRYZINSKI's theory [13] or from experimental data.

ion-electron and electron-ion interactions: Apart from radiative recombination we consider only elastic interactions between electrons and ions, which are readily derived from the general equations. The radiative recombination terms also enter into the collision integrals of the excited atomic components and have beeen examined there.

atom-photon interactions: The atom collision integrals involving photo-excitation, deexcitation, and ionization are described by the Eq. (45), (33) and no simplifications are possible. Only the radiative recombination terms can be reduced by the usual approximations for the electron gas. Considering the collision integrals for particle and total energy balance we have

$$G_i = \sum_{j<i} \left[B\overset{i}{\underset{\bar{\nu}j}{\uparrow}} u_{\bar{\nu}} n_j - \left(A\overset{i}{\underset{\bar{\nu}j}{\downarrow}} + B\overset{i}{\underset{\bar{\nu}j}{\downarrow}} u_{\bar{\nu}} \right) n_i \right] + \sum_{j>i} [\,..\,] + 8\, n_- n_+ \Omega^{(0)}_{-+\to\nu i} - \beta\overset{-+}{\underset{\nu i}{\uparrow}} n_i \,, \tag{77}$$

$$Q_i = \frac{m_-}{m} \, 8\, k\, T_- \left(1 - \frac{T}{T_-}\right)^2 n_- n_+ \Omega^{(1)}_{-+\to\nu i} + \left(\tfrac{3}{2}\, k\, T + \varepsilon_i\right) G_i \,. \tag{78}$$

VII. Results and Discussion

This paper provides on the basis of Grad's 8-moment approximation a system of equations describing all relevant collision and radiative processes in monatomic plasmas with the exception of the three body recombination. In particular, the general collision operators are formulated and expanded with respect to the moments. The binary particle processes — elastic or not — can be reduced to generalized Chapman-Cowling integrals. The radiative processes can be traced back to the Einstein and photoionization coefficients.

In its general form the theory is rather complicated. Fortunately a number of simplifications may be introduced based on plausible assumptions suggested through specific properties of the collision partners. Of specific success is the approximation that on the whole the energy of an electron after an inelastic collision is negligably small in comparison to its initial energy. As a consequence of this disposition all differential cross sections pass over into the total cross sections, quantities known, for instance, from Gryzinski's theory.

We feel that the above theory extends remarkably the description of a monatomic plasma in that it includes all processes except the three body recombination. Considering this generality the complication of the formalism should not surprise but rather be expected. With the simplifications of section VI the theory should be applicable to realistic systems.

The most stringent limitation of the model is the assumption of a near equilibrium electron distribution function together with the neglect of the three body recombination. Although this assumption is satisfied for certain systems, it may be incompatible for many others [14].

The applicability of the theory can be widely extended through the inclusion of the three body recombination. The formulation of the corresponding collision operator offers no problem — see, for instance the paper by LUDWIG and HEIL [2]. The problem is the incorporation of this collision operator within the formalism adopted here.

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